

# Implications of non-universality of soft terms in supersymmetric grand unified theories

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## Abstract

Most discussions of supersymmetric grand unified theories assume universality of the soft supersymmetry breaking terms at the grand scale. We point out that the behaviour of these theories might change significantly in the presence of non-universal soft terms. Particularly in  $SO(10)$ -like models with a large value of  $\tan\beta$  we observe a decisive change of predictions, allowing the presence of relatively light gauginos as well as small supersymmetric corrections to the b-quark mass. Some results remain rather stable, including the  $\mu$ - $M_{1/2}$  correlation. Models with small  $\tan\beta$  seem to be less affected by non-universality which mainly leads to the new possibility of small  $m_0$  (i.e. the squark and slepton soft mass parameter), excluded in the universal case.

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Properties of supersymmetric grand unified models (SUSY-GUT's) depend crucially on the values of the supersymmetry breaking terms. These terms include masses for the partners of the quarks and leptons, for the Higgs-bosons and the gauginos, as well as trilinear (A-terms) SUSY-violating interaction terms. The values of these terms cannot be understood within the framework of SUSY-GUT's; they are free input parameters to be chosen at the grand unified scale  $M_X$ . Thus soft terms are a signal of a more fundamental theory in which SUSY-GUT's should be embedded.

The simplest choice would correspond to universal soft terms at the GUT-scale. Usually one assumes universality of scalar masses  $m_0^2$ , gaugino masses  $M_{1/2}$  and A-parameters. The absence of flavour changing neutral currents puts constraints on the mass difference of squarks with the same charge. Universality of squark masses can be achieved through the choice of minimal kinetic terms in an underlying supergravity theory. It would imply that the non-gauge interactions have universal coupling strengths to all matter fields. In any case the choice of universal terms is an ad hoc assumption. Other choices might be equally well motivated and deserve further attention given the precision with which we can determine the gauge coupling constants.

There also exist theoretical arguments for a non-universality of the mass terms. In many supergravity models the Higgs superfields belong to a different sector than the matter superfields containing quarks and leptons. The Higgs multiplets might reside in sectors with  $N=2$  supersymmetry and are more closely related to the gauge multiplets than to the  $N=1$  chiral matter superfields. Since there are no restrictions on the Higgs masses from flavour changing neutral currents, one might consider the Higgs-mass as a separate input parameter that might differ from  $m_0^2$  of the matter fields. Moreover, explicit examples of orbifold string theories indicate the possibility of non-universal soft terms [1] and the same is true in general supergravity theories [2].

A second related question concerns the actual scale at which the (universal) terms are defined. In the underlying theory the fundamental scale (e.g. the Planck scale  $M_{Pl}$ ) might differ from  $M_X$ . Even in a framework of universal soft terms (most naturally chosen at  $M_{Pl}$ ) one would encounter non-universal behaviour at  $M_X$  [3]. Heavy threshold effects might strongly influence the spectrum of the soft terms at the very high scales.

Given this situation it seems to be mandatory in a discussion of SUSY-GUT's to work out explicitly the consequences of non-universal soft terms. At present we know a lot about SUSY-GUT's with universal terms [4]–[13]. If we include the assumption of Yukawa coupling unification and radiative electroweak symmetry breaking, we are in

many cases led to a very restrictive model with some definite predictions. We have then to investigate the stability of these predictions in the absence of universal mass terms. These are the questions we want to study in the present paper. Certain aspects of non-universal breaking terms have already been addressed in ref. [3, 14].

In our analysis we assume the Minimal Supersymmetric Standard Model (MSSM) particle spectrum below the unification scale and require:

- i) Exact gauge coupling unification at  $M_X$ .
- ii) Third generation Yukawa coupling unification: We will examine in turn the large and small  $\tan\beta$  regimes ( $\tan\beta = \frac{v_2}{v_1}$  is the ratio of the vevs of the two Higgs doublets), requiring in the first case that all three Yukawas unify (SO(10)-like unification), while in the second case that only the bottom and tau Yukawa couplings unify (SU(5)-like unification). Again exact unification at the scale  $M_X$  is assumed.
- iii) Radiative electroweak breaking: Starting with a symmetric theory at the unification scale, we require a proper breakdown of the SU(2) $\times$ U(1) electroweak symmetry induced through radiative corrections to the Higgs sector parameters.

In imposing gauge and Yukawa coupling unification we use the  $\overline{MS}$ -values [15]

$$\begin{aligned} \sin^2\theta_W &= 0.2324 - 0.92 \cdot 10^{-7} GeV^{-2} (M_t^2 - (143 GeV)^2) \pm 0.0003 , \\ \alpha_{em}^{-1} &= 127.9 \pm 0.1 , \end{aligned} \tag{1}$$

of the electroweak couplings at  $M_Z$  to determine  $M_X$  and  $\alpha_G$ , the unification scale and the gauge coupling at this scale respectively. The pole-mass value  $M_\tau=1.78 GeV$  [16] of the tau lepton is used to determine the common value  $h_b(M_X) = h_\tau(M_X)$  of the b- and  $\tau$ -Yukawa couplings at  $M_X$  respectively. In SO(10)-like unification schemes no further low energy input is needed to determine  $h_t(M_X)$ , the top Yukawa coupling at  $M_X$ , since it unifies with the other two. When we require SU(5)-like unification however, we use the bottom quark pole-mass  $4.7 GeV \leq M_b \leq 5.2 GeV$  [16] to fix the value of  $h_t$  at  $M_X$ . The value of  $\tan\beta$  is also an input in our analysis. Unification gives us a prediction for  $\alpha_3(M_Z)$ , the strong gauge coupling at  $M_Z$ , and  $M_t$ , the top quark pole-mass given a value of  $\tan\beta$ . In SO(10)-like scenarios we have in addition a prediction of the bottom quark mass<sup>1</sup>. In the determination of the running masses from the pole-mass data we neglect QED corrections, while taking QCD corrections into account at the two-loop level [17]. Renormalization below  $M_Z$  is carried out via two-loop QED and three-loop QCD renormalization group equations (RGE's) and the value of the strong gauge coupling used, is the one predicted from unification in each particular case that we study.

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<sup>1</sup>In these scenarios one could trade the prediction of  $M_b$  for that of the  $\tan\beta$  value.

To address the question of radiative electroweak symmetry breaking we start by assuming that low energy supersymmetry originates from a spontaneously broken supergravity theory. This theory is spontaneously broken by some unknown mechanism at the Planck or string scale, and the effect of this breakdown is parameterized by adding to the globally supersymmetric effective Lagrangian a series of soft breaking terms. This supersymmetry breaking part of the Lagrangian has the form:

$$\begin{aligned}
-\mathcal{L}_{soft} = & m_Q^2 |Q|^2 + m_U^2 |U|^2 + m_D^2 |D|^2 + m_L^2 |L|^2 + m_E^2 |E|^2 \\
& + m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 \\
& + \left[ h_t A_t Q H_2 \bar{U} + h_b A_b Q H_1 \bar{D} + h_\tau A_\tau L H_1 \bar{E} + \mu B H_1 H_2 + h.c \right] \\
& + \frac{1}{2} M_3 \bar{\lambda}_3 \lambda_3 + \frac{1}{2} M_2 \bar{\lambda}_2 \lambda_2 + \frac{1}{2} M_1 \bar{\lambda}_1 \lambda_1 ,
\end{aligned} \tag{2}$$

where  $m_i$ ,  $A_i$ ,  $B$ ,  $M_i$  are a set of parameters with mass dimension one,  $h_i$  are the Yukawa couplings and  $\mu$  is the coefficient of the Higgs mixing term in the superpotential. All fields appearing in (2) are scalar, except for the gauginos which are Majorana fermions. Generation indices have been suppressed.

From this Lagrangian one can extract the potential for the neutral components of the Higgs fields:

$$\begin{aligned}
V = & m_1^2 |H_1^0|^2 + m_2^2 |H_2^0|^2 - m_3^2 (H_1^0 H_2^0 + c.c) \\
& + \frac{\lambda_1}{2} |H_1^0|^4 + \frac{\lambda_2}{2} |H_2^0|^4 + (\lambda_3 + \lambda_4) |H_1^0|^2 |H_2^0|^2 ,
\end{aligned} \tag{3}$$

where  $m_1^2 \equiv m_{H_1}^2 + \mu^2$ ,  $m_2^2 \equiv m_{H_2}^2 + \mu^2$ , and we take  $m_3^2 \equiv B\mu$  to be positive. At scales where our theory is (softly broken) supersymmetric the quartic couplings  $\lambda_i$  satisfy the relations

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) , \quad \lambda_3 = \frac{1}{4}(g_2^2 - g_1^2) , \quad \lambda_4 = -\frac{1}{2}g_2^2 , \tag{4}$$

with  $g_1$ ,  $g_2$  being the U(1), SU(2) gauge couplings of the Standard Model respectively.

The parameters appearing in the Higgs potential (3) are evaluated at the unification scale and then renormalized down to the electroweak scale. The simplest choice of boundary conditions is the universal one

$$\begin{aligned}
m_{H_1}^2 = m_{H_2}^2 = m_Q^2 = m_U^2 = m_D^2 = \dots = m_E^2 = m_0^2 , \\
A_t = A_b = A_\tau = A , \\
M_3 = M_2 = M_1 = M_{1/2} ,
\end{aligned} \tag{5}$$

already extensively studied in the literature.

The aim of the present analysis is to investigate the phenomenological implications of non-universal soft breaking terms at the unification scale. We will mainly concentrate on two examples of non-universality, namely:

i) Disentangling the Higgs soft masses from the rest of the scalars:

$$\begin{aligned} m_{H_1}^2 &= m_{H_2}^2 = m_H^2 , \\ m_Q^2 &= \dots = m_E^2 = m_0^2 \end{aligned} \tag{6}$$

at  $M_X$ , with  $m_H$  being independent of  $m_0$ .

ii) Relaxing the universality within the Higgs sector:

$$m_{H_1}^2 \neq m_{H_2}^2 . \tag{7}$$

We in addition investigated the effect of non-universal gaugino masses and we will also briefly comment on the effect of lifting the universality between up- and down-type squarks.

We follow an up-down approach, renormalizing parameters from the unification scale down to low energies. Two-loop RGE's are used for the gauge and Yukawa couplings, one-loop for the soft mass parameters. In examining the breaking of the electroweak symmetry we minimize at  $M_Z$  the renormalization-group improved tree-level Higgs potential, and take into account the corrections coming from the mass splitting in the supermultiplets by decoupling heavy superpartners below some common scale  $M_S$ . The most sizeable corrections to the Higgs potential parameters come from the mass splitting in the quark and gluon supermultiplets. We therefore decouple the squarks and the gluino at the scale  $M_S$  and use below this scale the RGE's given in ref. [18], with the appropriate matching conditions for the Higgs potential parameters at the threshold [18, 5].  $M_S$  is chosen so as to minimize these threshold corrections and we require stability of our results against reasonable variations of  $M_S$  in the range of the squark masses. Between  $M_X$  and  $M_S$  the well-known MSSM RGE's are used (for easy reference see e.g. the appendix in ref. [5]).

In scanning the parameter space of the soft masses, we have excluded solutions in which the squarks are too heavy. For practical purposes a limit of  $\sim 2.5\text{TeV}$  has been imposed on the squark masses. We also impose the experimental lower bounds on the sparticle spectrum, the most relevant being a lower bound of approximately  $110\text{GeV}$  on the gluino and  $45\text{GeV}$  on the lightest stop.

A crucial parameter of the MSSM is  $\tan\beta$ . As we have already mentioned, the requirement of gauge and b- $\tau$  Yukawa coupling unification gives us a prediction of the top

quark pole-mass as a function of  $\tan\beta$  [19, 20]. For the top quark to be lighter than 200GeV (and indeed in the range recently reported by CDF [21]) two disjointed regimes for  $\tan\beta$  are allowed: a low regime with  $\tan\beta$  of the order of one and a high regime with  $\tan\beta$  between approximately 35 and 60. One should note that this prediction ignores corrections to the bottom and tau masses induced by heavy sparticle loops, which can be quite important when  $\tan\beta$  is large [22]. We will further comment on this point later on.

We performed our analysis for both  $\tan\beta$  regimes. When  $\tan\beta$  is large it can account for the mass hierarchy between the top and the bottom quark so the corresponding Yukawa couplings have similar values. It is possible then to assume that also  $h_t$  unifies with  $h_b$  and  $h_\tau$ , a scenario naturally realizable in the SO(10) unification scheme. On the other hand this similarity of the top and bottom Yukawa coupling values makes the breaking of the electroweak symmetry through radiative corrections difficult to achieve when  $\tan\beta$  is large. The reason for this is that the masses of the two Higgses evolve very similarly when there is no difference between  $h_t$  and  $h_b$ , so the condition  $m_1^2 m_2^2 < m_3^4$  for destabilizing the tree-level Higgs potential at the origin can only be fulfilled for some carefully chosen regions in the space of soft parameters when universality at  $M_X$  is assumed. This is not the case in the low  $\tan\beta$  regime where  $h_t \ll h_b$  so the product  $m_1^2 m_2^2$  can easily be driven negative in large regions of the parameter space, even if one starts with the two Higgs bosons degenerate at  $M_X$ . Therefore it should be noted that although non-universalities can exist in both scenarios, their role in the large  $\tan\beta$  scheme might be crucial for a more natural realization of radiative electroweak breaking, since they can induce the required hierarchy of the Higgs masses.

We start the presentation of our results with the large  $\tan\beta$  case. To be concrete we choose the specific value of  $\tan\beta=55$ , but the effects are qualitatively the same for the whole range, for which SO(10)-like unification is phenomenologically viable. For this value of  $\tan\beta$  the unification prediction for the strong gauge coupling is  $\alpha_3(M_Z) \simeq 0.130$ , for the top quark pole-mass  $M_t \simeq 192\text{GeV}$  and for the bottom pole-mass  $M_b \simeq 5.5\text{GeV}$ . These predictions are obtained if the supersymmetric RGE's are integrated all the way down to  $M_Z$ , while slightly lower values for  $\alpha_3(M_Z)$  and  $M_t$  ( $\sim 0.126$  and  $\sim 186\text{GeV}$  respectively) are predicted if one decouples the sparticles at an effective scale  $T_{SUSY}$  larger than  $M_Z$  [23, 19]. Note that the prediction for  $M_b$  is somehow larger than the upper bound quoted for this quantity ( $\sim 5.2\text{GeV}$ ). One should however be cautious, since for large  $\tan\beta$  the corrections to the bottom mass coming from heavy sparticle loops can be very big depending on the spectrum of the superpartners. We will address this issue in our discussion of the radiative electroweak symmetry breaking.

The dominant features observed in our analysis can be read off from figures 1 through 4. In each of these figures three different kinds of regions can be identified, corresponding to the following three cases studied:

**Case A** – Universal soft breaking terms: Depicted by the shaded regions enclosed in dotted lines.

**Case B** – Independent Higgs soft parameter: A Higgs soft parameter  $m_H$  independent of  $m_0$  has been introduced and the boundary conditions (6) are imposed at  $M_X$ . The solution points lie within the solid lines.

**Case C** – Non-degenerate Higgs doublets: The solution points enclosed by the dashed lines correspond to the following boundary conditions at the unification scale:

$$\begin{aligned} m_{H_1} &= m_0 , \\ m_{H_2} &= 0.8m_0 , \end{aligned} \tag{8}$$

where  $m_0$  is the common mass parameter for squarks and sleptons at  $M_X$ .

Our main conclusion from the study of non-universalities in the large  $\tan\beta$  regime is that radiative electroweak breaking and grand unification can be realized much more naturally, if one starts with non-universal soft masses within the Higgs sector. Aspects of this conclusion are present in all four figures presented. In Fig. 1 we show the  $m_0$ – $M_{1/2}$  plane. In the universal case it is known that proper symmetry breaking occurs only when  $m_0$  is smaller or at most of the order of  $M_{1/2}$ . A somehow simplified way to understand this is the following: When  $h_b \sim h_t$  the two main features that differentiate the running of the two Higgs mass parameters  $m_{H_1}^2, m_{H_2}^2$  are the asymmetric appearance of the up- and down-type squark mass parameters  $m_U^2, m_D^2$  in their RGE's, as well as the contribution of the tau Yukawa coupling to the equation for  $m_{H_1}^2$  only. Due to the larger hypercharge of the right-handed top compared to the right-handed bottom, large values of  $M_{1/2}$  tend to increase  $m_U^2$  with respect to  $m_D^2$  and so they indirectly induce an accelerated decrease of  $m_{H_2}^2$  compared to  $m_{H_1}^2$ . On the other hand large  $m_0$  values help  $m_{H_1}^2$  decrease faster than  $m_{H_2}^2$ , mainly because of the presence of the  $\tau$ -Yukawa term in the equation for  $m_{H_1}^2$ . Since both effects are more or less equally significant, an appropriate difference between  $m_{H_1}^2, m_{H_2}^2$  can only be achieved if  $M_{1/2}$  is larger than  $m_0$ . Furthermore there has to be a lower bound on  $M_{1/2}$ , since a minimum value of  $M_{1/2}$  is needed to make  $m_U^2$  sufficiently larger than  $m_D^2$ , or indirectly  $m_{H_2}^2$  sufficiently smaller than  $m_{H_1}^2$ .

Returning back to Fig. 1 then, one observes that disentangling the Higgs mass parameter  $m_H$  from  $m_0$  (case B), does not significantly change the universal picture:  $m_0$  and  $m_H$  must again be small for the  $\tau$ -term not to contribute significantly to the running

of the  $m_{H_i}$ 's,  $M_{1/2}$  must still be large to induce the right  $m_U^2 - m_D^2$  splitting which will lift the degeneracy of the Higgs doublets at low energies. If however this degeneracy is already lifted at the unification scale (case C), even by a fairly mild 20%, the solution space in the  $m_0 - M_{1/2}$  plane increases drastically:  $m_0$  need no longer be smaller than  $M_{1/2}$  and  $M_{1/2}$  is restricted from below only through the experimental bound on the gluino. These two effects have important consequences for the predicted sparticle spectrum and therefore also for the supersymmetric corrections to the bottom mass as we will see in the discussion of Fig. 2 and 4. In all three cases A, B and C very small values of  $m_0$  would result in the lightest stau being lighter than the lightest neutralino and we exclude these solutions by requiring the lightest supersymmetric particle (LSP) to carry no charge.

In Fig. 2 the  $M_{1/2} - \mu$  plane is depicted with  $\mu$  evaluated at  $M_Z$ . The tight correlation between  $M_{1/2}$  and  $\mu$  in the universal case has already been pointed out in ref. [10], where it has also been described with the help of semi-analytical formulas. A crude way to understand this correlation, at least for most of the solutions, is the following: At low energies both  $m_{H_1}^2$  and  $m_{H_2}^2$  are negative<sup>2</sup>, with  $m_{H_2}^2$  smaller than  $m_{H_1}^2$ . For a proper radiative breakdown  $\mu$  must then be such that  $m_1^2 > 0$  while  $m_2^2 < 0$  ( $m_i^2 = m_{H_i}^2 + \mu^2$ ). In other words  $\mu^2 = \mathcal{O}\left(-\frac{m_{H_1}^2 + m_{H_2}^2}{2}\right)$  (the difference of  $m_{H_1}^2$ ,  $m_{H_2}^2$  is in general much smaller than their absolute values). Now raising  $M_{1/2}$  has the effect of driving  $m_{H_1}^2$ ,  $m_{H_2}^2$  to lower values<sup>3</sup>, so we need larger values of  $\mu$  to fulfill the symmetry breaking conditions. The dependence of  $\mu$  on  $M_{1/2}$  turns out to be practically linear in the universal case for the parameter space scanned. The lower limit on  $M_{1/2}$  has been explained in the discussion of Fig. 1. Fig. 2 confirms what has already been observed in Fig. 1: The effect of treating the Higgses independently (case B) is minimal, preserving, though slightly loosening the linear  $M_{1/2} - \mu$  correlation. However, when one lifts the mass-degeneracy of the Higgses at  $M_X$  (case C), a much larger area of the  $\mu - M_{1/2}$  plane gives proper radiative breakdown. The correlation between  $\mu$  and  $M_{1/2}$  is in this case still valid, since it remains true that raising  $M_{1/2}$  drives the  $m_{H_i}^2$ 's smaller, thus requiring larger  $\mu$  for proper symmetry breaking. However, since in this case also large values of  $m_0$  are permitted,  $m_0$  contributes significantly in the running of the  $m_{H_i}^2$ 's when  $M_{1/2}$  is small (it is  $m_0$  that makes the squarks heavy when  $M_{1/2}$  is small). For this reason the dependence of  $\mu$  on  $M_{1/2}$  clearly deviates from linearity in the lower part of the  $\mu - M_{1/2}$  plane. This has the effect that even for very small values of  $M_{1/2}$ ,  $\mu$  still remains fairly large.

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<sup>2</sup>The Yukawa terms in their RGE's dominate over the gauge terms.

<sup>3</sup>Larger  $M_{1/2}$  means heavier squarks and therefore larger contributions from the Yukawa terms. The gauge terms do not compensate these contributions because they contain only the electroweak sector.



Let us now turn to some characteristic features of the predicted sparticle spectrum. An important observation is that radiative electroweak breaking requires  $\mu$  to be always significantly larger than  $M_{1/2}$ . This has the effect that the mixing of the neutralinos and the charginos is small and therefore the lightest neutralino and the lightest chargino are an almost pure bino and an almost pure wino respectively. In Fig. 3 we plot the lightest chargino tree-level mass versus  $\mu$  for the three cases A,B and C. A noteworthy feature is that certain non-universalities (e.g. case C) allow for fairly light chargino masses, whereas with universal soft terms there is a lower bound of  $\sim 300\text{GeV}$  for the lightest chargino. The same applies for the lightest neutralino which can be as light as  $\sim 45\text{GeV}$  in case C while in the universal case it is always heavier than  $\sim 150\text{GeV}$ . These observations are of course directly related to the lower bound on  $M_{1/2}$  in the various cases.

Closely related to the predicted sparticle spectrum are the corrections to the bottom mass coming from sparticle loops. The dominant corrections come from sbottom-gluino and stop-chargino loops [22]. Defining the running bottom mass by  $m_b = h_b v_1 (1 + \delta m_b)$ , the magnitude of the corrections is given by the approximate formula [10]:

$$\delta m_b = \frac{2\alpha_3}{3\pi} K_1 \tan\beta \frac{M_{\tilde{g}} \mu}{m_{max1}^2} + \frac{h_t^2}{16\pi^2} K_2 \tan\beta \frac{A_t \mu}{m_{max2}^2} , \quad (9)$$

where  $K_1, K_2$  are coefficients of order one,  $M_{\tilde{g}}$  is the gluino mass and  $m_{max1}^2, m_{max2}^2$  the squared masses of the heaviest particles running in the corresponding loops. We evaluate the corrections at the electroweak scale. Both corrections are proportional to  $\tan\beta$  and therefore can be very large if the mass ratios appearing in (9) are not small. It is clear from Fig. 2 that  $M_{\tilde{g}}$  and  $\mu$  are closely correlated ( $M_{\tilde{g}} = \frac{\alpha_3}{\alpha_G} M_{1/2} \simeq 3M_{1/2}$  at  $M_Z$ ). Also since the running of  $A_t$  is very similar to that of  $M_{\tilde{g}}$ ,  $A_t$  is always roughly of the order of  $M_{\tilde{g}}$ . In the universal scenario  $M_{\tilde{g}}$  is very heavy because of the lower limit on  $M_{1/2}$ , so all three quantities  $M_{\tilde{g}}, \mu$  and  $A_t$  are large. The heaviest particles running in the two relevant loops are in this scenario the gluino and the heavy stop respectively. Since however also the heavy squarks are of the order of the gluino mass, the ratios appearing in (9) are  $\mathcal{O}(1)$  and the corrections to the bottom mass are not suppressed. The situation is the same if one disentangles the Higgses from squarks and sleptons, since the predicted spectrum is very much like the one of the universal scenario. The possibility of suppressing the corrections arises if one considers case C. The fact that very low values for  $M_{1/2}$  are allowed in this case, while at the same time  $m_0$  can be large (see Fig. 1), gives solutions with squarks quite heavier than  $M_{\tilde{g}}$  and  $A_t$ . This means small ratios in (9) and therefore suppressed corrections.

The above remarks are reflected in Fig. 4. For the cases A and B the supersymmet-

ric corrections to the bottom mass are in all solutions of the order of 30%, positive or negative depending on the sign of  $\mu$ . So in these cases the prediction  $M_b=5.5\text{GeV}$  for the uncorrected mass is modified to  $M_b=4.2\text{GeV}$  or  $M_b=6.9\text{GeV}$  (depending on the sign) when corrections are taken into account. Both value lie outside the range quoted for the b pole-mass. In case C the situation is different: There are solutions covering the whole range of corrections between 5% and 60%. So although also in this case very large corrections are not excluded, the solutions with  $M_{1/2}$  small provide the possibility of suppressing them and successfully predicting the bottom quark mass: Negative corrections between 15% and 5% would give  $4.7\text{GeV} \leq M_b \leq 5.2\text{GeV}$ .

Until now we have concentrated on non-universalities directly related to the Higgs sector, a natural thing to do if one wants to study the radiative breaking of the electroweak symmetry. However non-universalities introduced in other sectors of the theory can also affect the process of symmetry breaking, since these sectors couple to the Higgs sector and therefore affect the running of the Higgs potential parameters. Relaxing the universality within the gaugino sector at  $M_X$  causes negligible modifications of the solution space. This is so because essentially only  $M_1$  plays a decisive role in the process of symmetry breaking (it differentiates the running of  $m_U^2$ ,  $m_D^2$  and thus also that of  $m_{H_i}^2$ ). So if the  $M_i$ 's are not universal at  $M_X$  then  $M_1(M_X)$  plays the role that  $M_{1/2}$  plays in the universal case, at least as far as the symmetry breaking is concerned. However, lifting the universality between up- and down- type squarks at  $M_X$  has relevant effects<sup>4</sup>, since  $m_U^2$  and  $m_D^2$  induce a non-uniform evolution of the  $m_{H_i}^2$ 's, as has previously been explained. This is an indirect way to make the two Higgses non-degenerate, a fact also reflected in the solution space of such scenarios which looks very similar to that of case C. It should however be noted that because of the indirect way the non-degeneracy is induced, a mass splitting of the order of 40% between up- and down-type squarks is needed, in order to significantly affect the process of radiative electroweak breaking. Mass splittings of the order of 20% only reproduce the solution space of the universal scenario.

Let us finally turn to the low  $\tan\beta$  regime. We will present illustrative results for  $\tan\beta=1.8$ , for which unification predicts the strong gauge coupling to be  $\alpha_3(M_Z)=0.129$  and the top pole-mass  $M_t=187\text{GeV}$ , when  $T_{SUSY} = M_Z$  (for  $T_{SUSY}=500\text{GeV}$  one finds  $\alpha_3(M_Z) \simeq 0.123$  and  $M_t \simeq 185\text{GeV}$ ).

In Fig. 5 we show the  $m_0$ - $\mu$  plane with the three types of regions corresponding again to the cases A,B and C defined previously. Let us first consider more closely the universal

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<sup>4</sup>We thank S. Pokorski for discussions on this point

scenario. When  $\tan\beta$  is small there is a large hierarchy between  $h_t$  and  $h_b$  (and between  $h_t$  and  $h_\tau$ , of course, but we ignore the latter in the following discussion since its contribution is very small). Moreover the requirement of  $b$ - $\tau$  unification is only fulfilled for very large values of  $h_t$  [24, 19, 20], which means that at low energies  $h_t$  is always very close to its infrared quasi-fixed point value [25]. These two facts make  $h_t$  dominate in the process of radiative electroweak symmetry breaking when  $\tan\beta$  is small. This has as a consequence that if one wants to drive  $m_{H_2}^2$  smaller than  $m_{H_1}^2$  both  $M_{1/2}$  and  $m_0$  contribute in the right direction. For  $M_{1/2}$  the reasons given in the large  $\tan\beta$  discussion apply here as well (it makes  $m_U^2 > m_D^2$ )<sup>5</sup>. Now however also  $m_0$  tends to make  $m_{H_2}^2$  smaller than  $m_{H_1}^2$  and not vice versa as was the case for large  $\tan\beta$ . This is again the effect of the Yukawa terms and the fact that  $h_b, h_\tau$  are now negligible compared to  $h_t$ . One further important feature when  $\tan\beta$  is small is that large values of  $M_{1/2}$  drive certain squarks very heavy. This is so because of the  $h_b$  term in the  $m_D^2$  equation, which tends to decrease the value of  $m_D$ , has a vanishing contribution. Therefore, by setting an upper bound of 2.5TeV on the squark masses, we effectively forbid large values for the  $M_{1/2}$  parameter, in particular when  $m_0$  is large. This means that for most of the solutions  $m_0$  is larger than  $M_{1/2}$  and dominates in the process of symmetry breaking. This explains the strong correlation between  $m_0$  and  $\mu$ . Also the lower bound on  $m_0$ , present in Fig. 5 in the universal case, is the effect of setting an upper bound of 2.5TeV on the squark masses: If  $m_0$  were to be vanishingly small, a fairly large value of  $M_{1/2}$  would be needed to induce the splitting of the Higgs doublets. Such a value would however drive certain squarks unacceptably heavy, and this is the reason why solutions with very small  $m_0$  have been excluded. One should however note that if very heavy squarks were not considered to be a problem, symmetry breaking alone does not forbid vanishing  $m_0$ .

Returning then to Fig. 5 let us see how non-universalities affect the above picture. It is now case C (non-degenerate Higgses at  $M_X$ ) that has negligible effects on the universal results: The hierarchy of the  $m_{H_i}^2$ 's can be easily induced radiatively, non-universal initial conditions do not enlarge the solution space. However, if one reserves  $m_0$  for squarks and sleptons only and treats the Higgses independently (case B), there are observable effects in the  $m_0$ - $\mu$  plane: Solutions with very small values of  $m_0$  are allowed. The reason for this is that  $m_H$  can in this case compensate for the vanishing contribution of  $m_0$  in the running of the  $m_{H_i}^2$ 's, so  $M_{1/2}$  does not have to be so large that squarks become too heavy.

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<sup>5</sup>In fact, since for small  $\tan\beta$  the Yukawa terms in the equation for  $m_{H_1}^2$  are negligible, the splitting of  $m_U^2, m_D^2$  is no longer crucial: making squarks heavy is enough to accelerate the decrease of  $m_{H_2}^2$  and this is the actual role of  $M_{1/2}$  in the low  $\tan\beta$  regime.

In short we can have vanishing  $m_0$  with squarks below 2.5TeV.

We thus have seen that many predictions of conventional SUSY-GUT's are changed in the presence of non-universal soft breaking terms. The importance of this non-universality is most prominent in SO(10)-like models with large  $\tan\beta$ . There some of the main conclusions of the universal case have to be revised. Small values of  $M_{1/2}$  are now allowed bringing back the lightest charginos and neutralinos in the experimentally accessible mass range. The presence of these light particles in the large  $\tan\beta$  regime might also have consequences for a discussion of the supersymmetric corrections to the  $Zb\bar{b}$ -vertex. The presence of non-universal terms is crucial for the size of radiative corrections to the b-quark mass from supersymmetric particles. While in the universal case these corrections are always large (when  $\tan\beta$  is large), they might be reduced in the non-universal case.

Fortunately, however, there are many situations where the effects of non-universal terms are less important. In the presence of large  $M_{1/2}$  this happens very often since in that case the radiative corrections to the soft masses tend to wash out any primeval non-universality. The case of small  $\tan\beta$  is less affected by non-universality, as could have been expected. Here a split of the two Higgs masses is not so important, since radiative symmetry breakdown can be achieved for a wide region of the (universal) parameter space. On the other hand a situation where the Higgs masses are split from squark and slepton masses, leads to the new possibility of small  $m_0$ , excluded in the universal case. Otherwise the small  $\tan\beta$  results for the universal case are pretty stable. Even in the case of large  $\tan\beta$  some predictions remain rather insensitive to non-universalities. In particular we note the the stability of the  $\mu$ - $M_{1/2}$  correlation as shown in Fig. 2.

Thus many results of the universal case might have more general validity. Nonetheless one should be aware of the fact, that in a given model the specific predictions should be carefully worked out. Especially in models with large  $\tan\beta$  we find large sensitivity to a non-universality of the soft terms that might have decisive influence on the phenomenological properties.

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## Figure Captions

Figure 1: The  $m_0$ – $M_{1/2}$  plane (in GeV units) for  $\tan\beta=55$  and SO(10)–like unification. The solution space for case A (universal terms) is the shaded region enclosed in the dotted line, for case B ( $m_H \neq m_0$ ) the region contained in the solid line and for case C ( $m_{H_1} = m_0$ ,  $m_{H_2} = 0.8m_0$ ) the region above the dashed line.

Figure 2: The  $M_{1/2}$ – $\mu$  plane (in GeV units) also for  $\tan\beta=55$  and SO(10)–like unification. Again case A corresponds to the shaded regions within the dotted lines, case B to the regions bounded by the solid lines and case C to the regions inside the dashed lines. The values for the  $\mu$  parameter shown are those at  $M_Z$ .

Figure 3: The lightest tree–level chargino mass  $m_{char}$  is plotted against  $\mu$  (in GeV units) for  $\tan\beta=55$  and SO(10)–like unification. Since the lightest chargino is an almost pure wino  $m_{char} \simeq M_2 \simeq 0.8M_{1/2}$ , so this plot looks very similar to that of Fig. 2. The assignment of cases and regions is the same as in the previous figures and  $\mu$  is again shown with its low energy values.

Figure 4: The supersymmetric corrections  $\delta m_b$  (in %) are plotted against  $\mu$  (in GeV) for  $\tan\beta=55$  and SO(10)–like unification. The corrections are defined by  $m_b = h_b v_1 (1 + \delta m_b)$  and are evaluated at  $M_Z$ . Case–region assignments as before,  $\mu$  shown with its values at  $M_Z$ .

Figure 5: The  $m_0$ – $\mu$  plane (in GeV units) for  $\tan\beta=1.8$  and SU(5)–like unification. The solutions giving proper symmetry breaking are for case A (universal terms) bounded by the dotted lines, for case B ( $m_H \neq m_0$ ) by the solid lines and for case C ( $m_{H_1} = m_0$ ,  $m_{H_2} = 0.8m_0$ ) by the dashed lines. The  $\mu$  parameter is evaluated at  $M_Z$ .

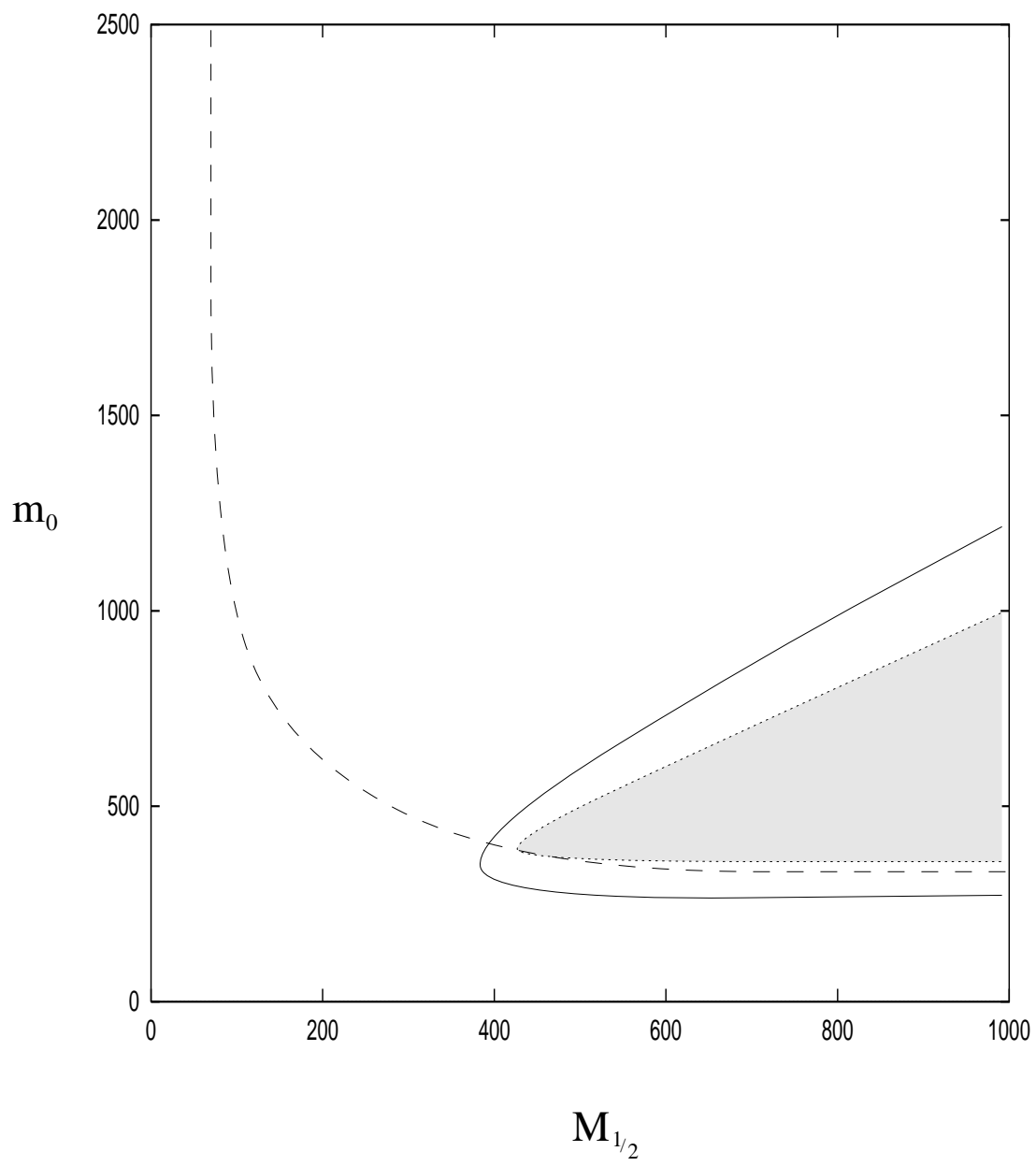


Figure 1



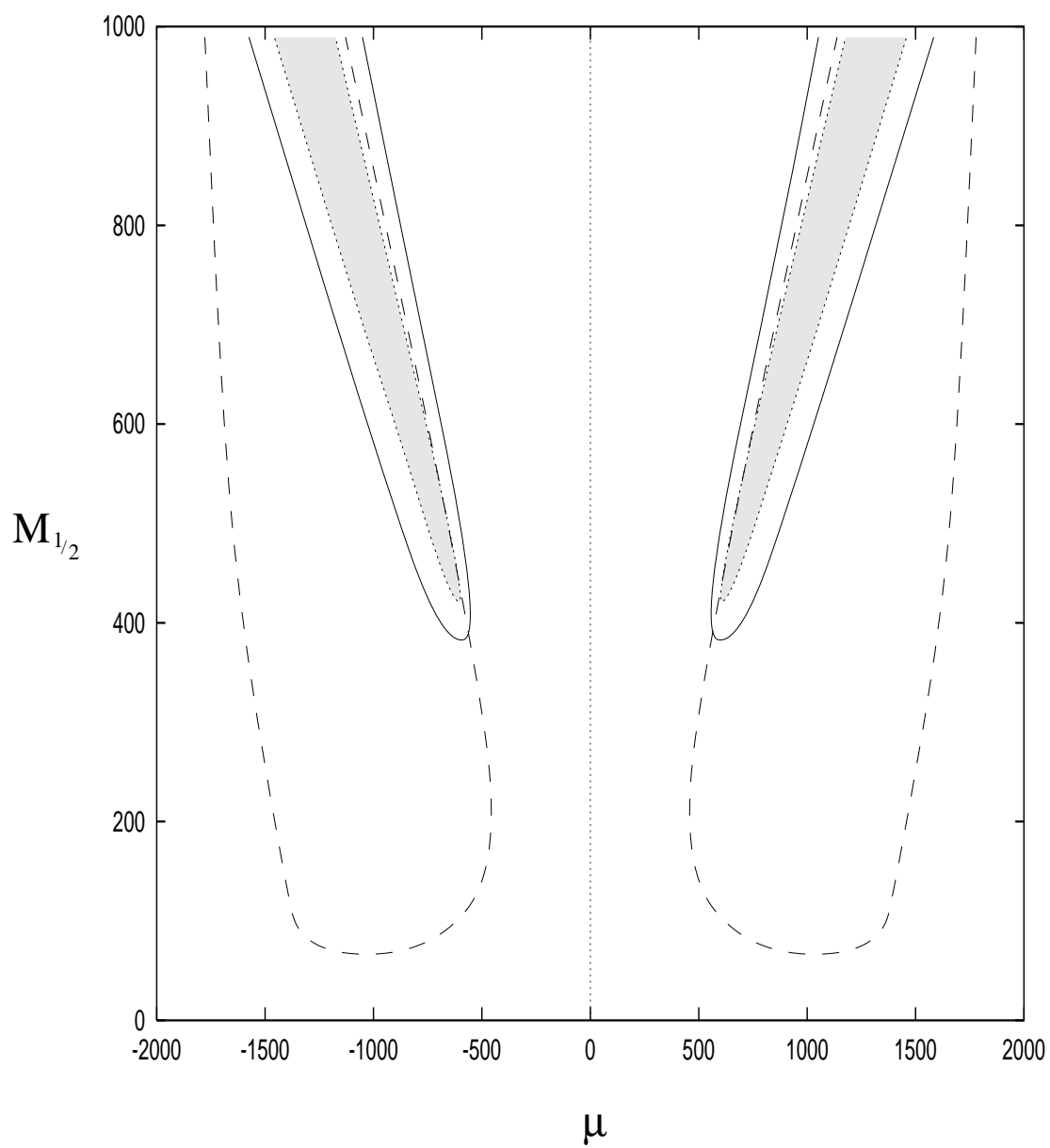


Figure 2

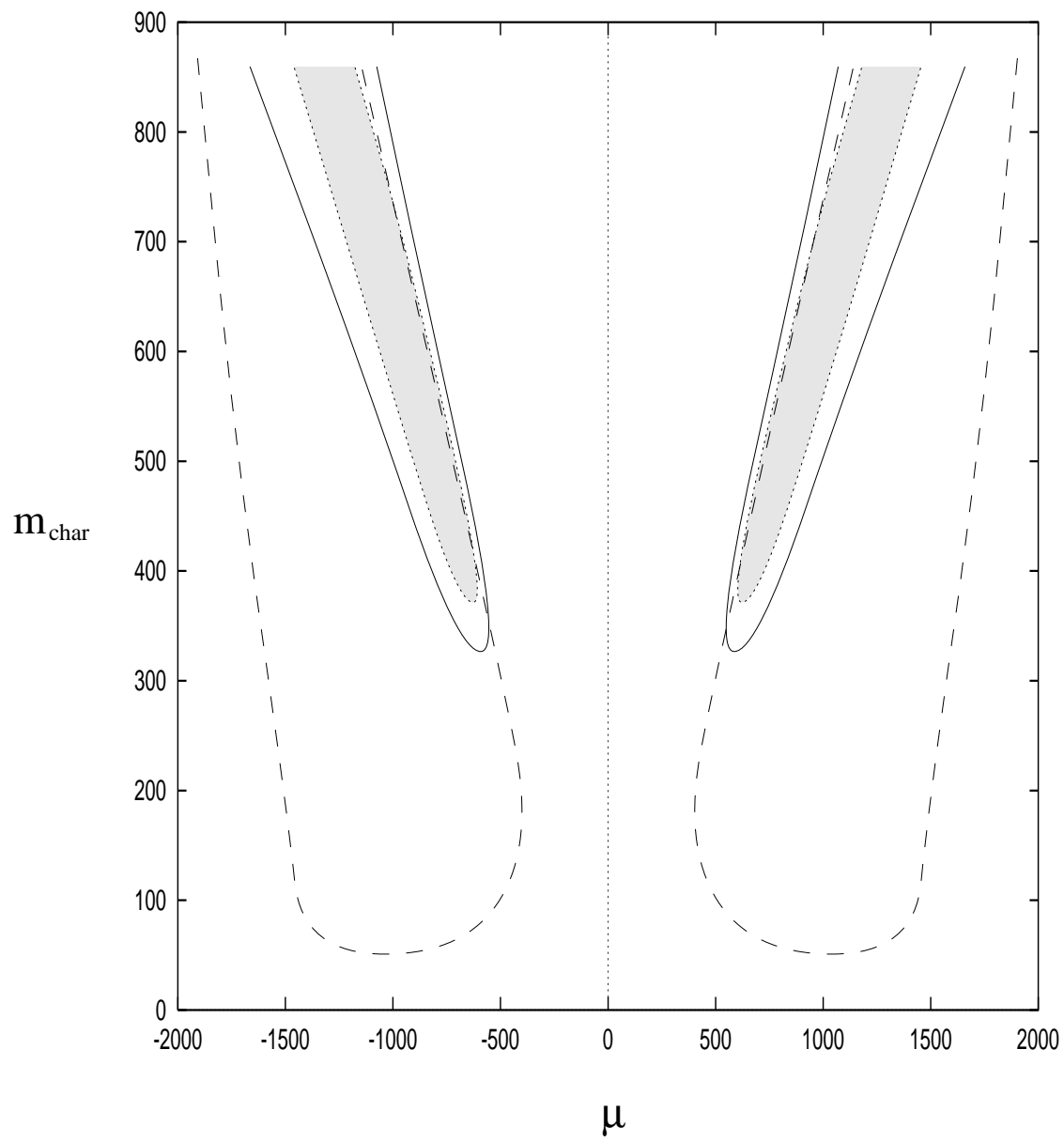


Figure 3

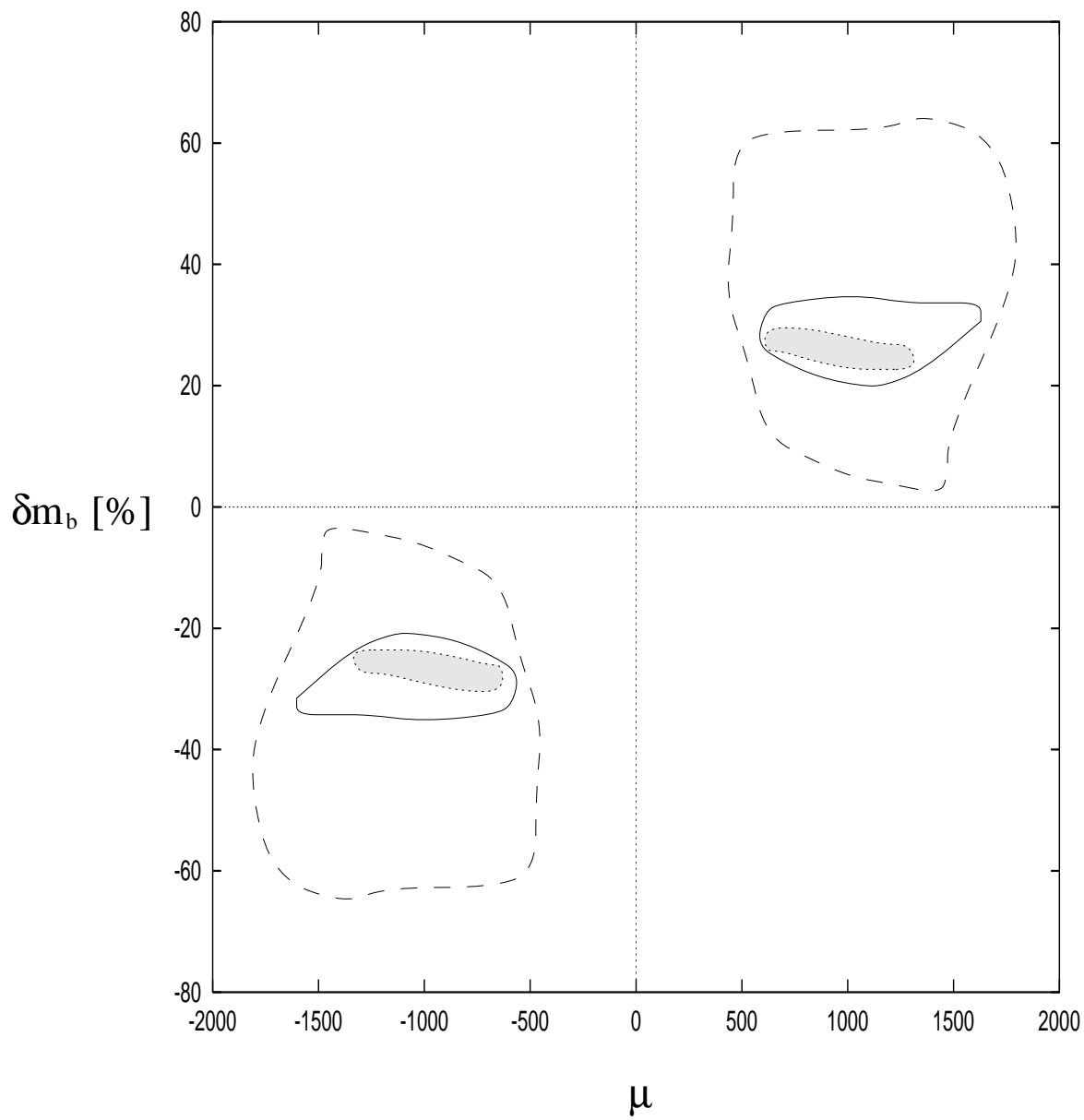


Figure 4

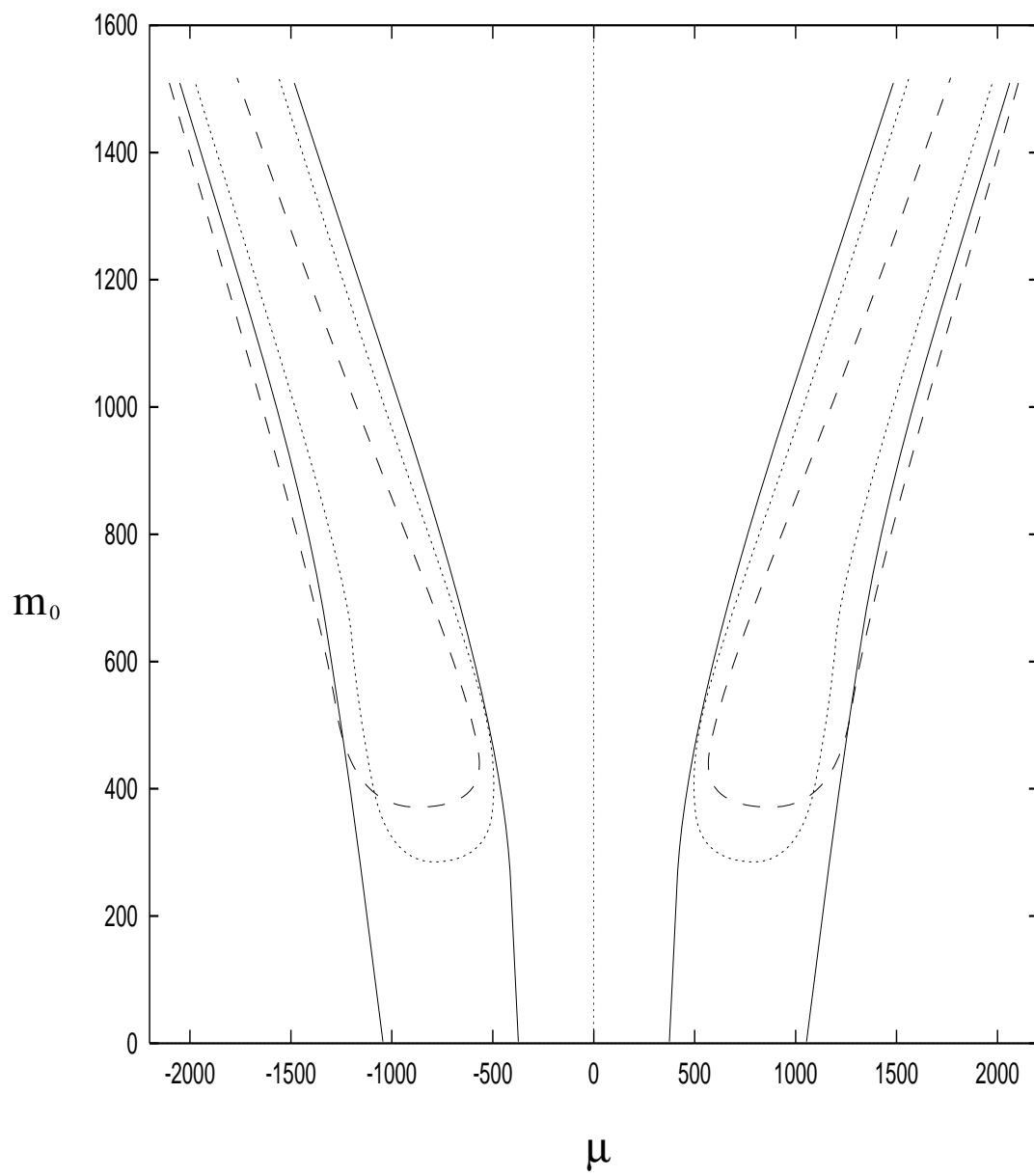


Figure 5

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